Spin versus helicity

in processes involving transversity

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We construct the spin formalism in order to deal in a direct and natural way with processes involving transversity which are now of increasing popularity. The helicity formalism which is more appropriate for collision processes of definite helicity has been so far used also to manage processes with transversity, but at the price of computing numerous helicity amplitudes generally involving unnecessary kinematical variables.In a second step we work out the correspondence between both formalisms and retrieve in another way all results of the helicity formalism but in simpler forms.We then compute certain processes for comparison.A special process:the quark dipole magnetic moment is shown to be exclusively treated within the spin formalism as it is directly related to the transverse spin of the quark inside the baryon.

1. Motivating the spin

The amplitude of a process involving fermions of spin one half is generally written as

$$\bar{u}(k,s)...X.....u(k',s') \tag{1}$$

where $u(k,s)$ is the part of the wave function which describes the spin one half of a particle of energy-momentum p and spin projection s and where ellipses indicate other Dirac spinors. The probability for a given process to occur is the squared modulus of the amplitude. It is usually expressed as a trace over spinor indices by use of standard projectors

$$Tr(\bar{X}\frac{\not{p}'+m}{2m}\frac{1+\gamma_5\not{s}'}{2}\cdots X\cdots\frac{\not{p}+m}{2m}\frac{1+\gamma_5\not{s}}{2}) \tag{2}$$

The trace form of the probability is compact, Lorentz invariant and, in addition offers the possibility to handle the case of several γ- matrices amplitudes (several loops) using machine facilities. Several symbolic programs are made available for such symbolic computations. In the next section we will show that amplitudes as well are re-expressed as traces similarly to probabilities, and hence benefit from the same computational facilities . There exits a work similar to the present one but uses the concept of helicity(helicity formalism HF)[1] but not the concept of the spin (spin formalism SF) . Here we construct the SF and show it to be equivalent to the HF . But why the spin? if the helicity did all as we know. The answer is that the HF deals naturally with processes where the states are helicity states. It also deal with processes with states polarized along arbitrarily directions not necessary along the momenta, but at the price of computing several helicity amplitudes and using unnecessary intermediary kinematical variables . To see this, we anticipate some formulas . Let us for instance compute the electromagnetic vertex, where the initial and final particles are polarized along the transverse directions to their momenta. The vertex amplitude is of the form

$$Tr(\rho_{-ss}\gamma_\mu) \tag{3}$$

This is a one-step computation. The other way to compute the spin vertex is to use its decomposition on the helicity vertices

$$Tr(\rho_{-s,s}\gamma_\mu) = -\frac{i}{2}\left[s(Tr(\rho_{1,1}\gamma_\mu) + Tr(\rho_{-1,-1}\gamma_\mu)) + Tr(\rho_{1,-1}\gamma_\mu) + Tr(\rho_{-1,1}\gamma_\mu)\right] \tag{4}$$

Therefore the other alternative is to compute the four helicity amplitude (the right hand side of(4)). So treating transverse (or general) polarizations by the HF amounts rewriting such polarizations in terms of helicity states according to(4) which complicates the analysis somehow. On the other hand processes with transverse polarizations are no longer "academic" processes since our revival of the transversity [2] in 1992 in which we showed the relevance of the concept of transversity in hadronic physics.

A physical example (not the computation of the vertex above) where the HF is systematically used to deal with tranversity is the one- spin transverse asymmetries . The difference of the differential cross sections are related to helicity amplitude as[3]

$$\begin{aligned} d\sigma_\uparrow - d\sigma_\downarrow &= |<\cdots|T\,|\uparrow>\,|^2 - |<\cdots|T\,|\downarrow>\,|^2 \\ &= 2Im\left\{<\cdots|T\,|+><+|T\,|\cdots>\right\} \end{aligned} \tag{5}$$

where $|+>$and$|->$ stand for the helicity states $|\pm 1/2>$ and $|\uparrow>$ and $|\downarrow>$ for the states polarized along the transverse direction $\pm\hat{x}$. Needless

to say that it is simpler to compute the first line in 5 which implies computing the spin amplitude $< \cdots |T\,|\uparrow\, or \downarrow>$ if we can (That is the aim of the SF)than the second line which implies computing the product of two helicity amplitudes$< \cdots |T\,|+><+|T\,|\cdots>$ a much more complex spin structure.
It is the aim of the present study to give the full expression for $\rho_{ss'}(k,k')$ which is the cornerstone of the SF which is constructed here to handle amplitudes of general polarizations . A helicity approach to a process with general polarizations not only is non appropriate but is still more cumbersome when more particles in the process are spinning, as this enhances the number of helicity components to be computed: the scattering amplitude from the generally polarized state of the electron-positron system to a generally polarized final state of say two photons will involve sixteen helicity amplitudes $T^{h\bar{h}}_{\lambda\bar{\lambda}}$ with $h,\bar{h}$ the helicity of the photons. Even if some symmetries are present such as chirality or parity (both often absent in supersymmetic models for instance) it will not reduce the number of helicity amplitudes notably. Another issue which necessitates, but this time, an exclusive treatment within the SF (as it depends directly on the transverse spin)is the quark dipole magnetic moment which can ultimately be written as (see subsection 4-2)

$$\sim \int |\vec{k}|^2 \vec{s}_\perp(k) \frac{d^3k}{(2\pi)^3} \tag{6}$$

The above considerations all together show definitely that the prejudice against the use of spin (usually considered as unessential concept as compared to the helicity) has no raison d'etre, and this is sufficient to motivate us to construct the SF

2. The amplitude as a trace

We propose in this paper, among other things to rewrite the probability amplitude as a trace over spinor indices, as in(2). In this way the amplitude gets a compact form in terms of “generalized” projectors, thus becoming suitable for analysis and ready for symbolic computations. To this end we rewrite (1) as (summation over repeated indices is understood).

$$\begin{array}{l} \bar{u}_\alpha(k,s)\cdots f_{\alpha\alpha'}\cdots u_{\alpha'}(k',s') \\ = \cdots f_{\alpha\alpha'}\cdots u_{\alpha'}(k',s')\bar{u}_\alpha(k,s) \\ = Tr(\cdots f\rho) \\ \rho_{\alpha'\alpha}(k',s',k,s) = u_{\alpha'}(k',s')\bar{u}_\alpha(k,s) \end{array} \tag{7}$$

The trace involves the 4 by 4 generalized spin density matrix $\rho_{\alpha'\alpha}(k', s', k, s)$ which when $k' = k$ and s$' = s$ reduces to the standard projector

$$(\frac{\not{k} + m}{2m})(\frac{1 + s\gamma^5 \not{s}}{2}) \tag{8}$$

The approach we follow to work out the expression for the projector ρ is to extract it from the primitive form it has at the rest frame (relatively easy to compute)by performing a Lorentz boost. The form of the generalized density is (from now on we hide Dirac indices)

$$\rho(k', s', k, s) = \frac{2\mathcal{N}_{s's}}{1 + ss'\vec{\zeta}.\vec{\zeta}'}(\frac{\not{k}' + m'}{2m'})(\frac{1 + s'\gamma^5 \not{s}'}{2})\Re(\vec{k}', \vec{k})(\frac{\not{k} + m}{2m})(\frac{1 + s\gamma^5 \not{s}}{2}) \tag{9}$$

The operator $\Re(\vec{k}', \vec{k})$ flips the momentum from $\vec{k}'$ to $\vec{k}$ while $\mathcal{N}_{s's}$ a matrix in the two dimensional space of solutions ($s = \pm 1$is twice the spin) is responsible for the spin flip making the passage from the state $u_\alpha(k', s')$to the state $u_\alpha(k, s)$computed in the rest frame The matrix $\Re$ is shown to have the explicit form

$$\begin{aligned} \Re(\vec{k}', \vec{k}) &= \exp(-\tfrac{\omega'}{2}\gamma_0 \tfrac{\vec{\gamma}.\vec{k}'}{|\vec{k}'|}) \exp(\tfrac{\omega}{2}\gamma_0 \tfrac{\vec{\gamma}.\vec{k}}{|\vec{k}|}) \\ &= \cosh(\tfrac{\omega'}{2}) \cosh(\tfrac{\omega}{2}) \\ &\quad - \sinh(\tfrac{\omega'}{2}) \cosh(\tfrac{\omega}{2})\gamma_0 \tfrac{\vec{\gamma}.\vec{k}'}{|\vec{k}'|} + \sinh(\tfrac{\omega}{2}) \cosh(\tfrac{\omega'}{2})\gamma_0 \tfrac{\vec{\gamma}.\vec{k}}{|\vec{k}|} \\ &\quad + \sin h(\tfrac{\omega'}{2}) \sin h(\tfrac{\omega}{2}) \tfrac{\vec{\gamma}.\vec{k}'}{|\vec{k}'|} \tfrac{\vec{\gamma}.\vec{k}}{|\vec{k}|} \end{aligned} \tag{10}$$

with $\omega = -\tanh^{-1}(\frac{|\vec{k}|}{k^0})$ and idem for ω'. At this point, further simplifications are possible and we get the final simple result

$$\Re = \sqrt{\frac{mm'}{(k_0 + m)(k'_0 + m')}}(\gamma_0 + 1) \tag{11}$$

3. principal formulas of the HF and the SF

we recapitulate here the principal results of the SF and the HF written in the center of mass frame. The ortho-normal frame $(\vec{k}, \vec{\eta}_1, \vec{\eta}_2)$ will be used .

$$\begin{aligned} &\vec{k} = |\vec{k}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ &\eta_1 = (0, \vec{\eta}_1) = (0, \cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ &\eta_2 = (0, \vec{\eta}_2) = (0, -\sin\phi, \cos\phi, 0) \end{aligned} \tag{12}$$

The HF formulas are

$$\begin{array}{rcl}\rho_{\lambda,-\lambda} & = & u(k',\lambda)\bar{u}(k,-\lambda) = -\lambda\gamma_0\frac{\not k+m}{2m}\frac{1-\lambda\gamma^5\not\eta_3}{2}e^{i\lambda\phi} \\ \rho_{\lambda,\lambda} & = & u(k',\lambda)\bar{u}(k,\lambda) = -\lambda\gamma_0\frac{\not k+m}{2m}\gamma_5\frac{\not\eta^\lambda}{2}e^{i\lambda\phi} \\ \not\eta^\lambda & = & \not\eta_1 - i\lambda\not\eta_2\end{array} \quad (13)$$

and the SF in our conventions are

$$\begin{array}{rcl}\rho_{s,s}|_{\vec{k'}} & = & u(k',s)\bar{u}(k,s)|_{\vec{k'}} = -i\gamma_0(\frac{\not k+m}{2m})(\frac{1-s\gamma^5\not s_L}{2}) \\ \rho_{-s,s}|_{\vec{\zeta}} & = & u(k',-s)\bar{u}(k,s)|_{\vec{\zeta}} = -\gamma_0(\frac{\not k+m}{2m})(\frac{1+s\gamma^5\not s_\perp}{2})\end{array} \quad (14)$$

and the links between both formalism are

$$\begin{array}{rcl}\rho_{s,s}|_{\vec{k'}} & = & \rho_{\lambda,-\lambda} \\ is(\Delta(\vec{\eta'_1},s) - i\lambda\Delta(\vec{\eta'_2},s)) & = & u(k',\lambda)\bar{u}(k,\lambda) \\ \Delta(\vec{\zeta},s) & = & \\ u(k',-s)\bar{u}(k,s)|_{\vec{\zeta}=\vec{\eta}_1,\vec{\eta}_2} - \frac{1}{2}\sum_{s=\pm 1} u(k',-s)\bar{u}(k,s)|_{\vec{\zeta}=\vec{\eta}_1,\vec{\eta}_2} & & \end{array} \quad (15)$$

4. Computing a given process in HF and in SF

4.1. $e^- \to e^- + \gamma$*and* $e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$

Let us compute the simplest possible amplitude in the SF and in the HF as an illustration of the performance of the spin over the helicity in the presence of a general polarization. So a vertex such as $e^- \to e^- + \gamma$or even simpler such as $e^- \to \tilde{e} + \tilde{\gamma}$ where $(\tilde{e}, \tilde{\gamma})$ is a (slectron, photino) system, may serve as an example of such compuations. The decomposition of the spin state over the helicity states is equivalent to the spin projector ρ_{-ss} decomposed over the helicity projectors

$$\rho_{-s,s} = -\frac{i}{2}\left[s(\rho_{1,1} + \rho_{-1,-1}) + \rho_{1,-1} + \rho_{-1,1}\right] \quad (16)$$

The electromagnetic vertex is simply the trace

$$Tr(\rho_{-ss}\gamma_\mu) \quad (17)$$

If we chose to compute the vertex in the SF, this single trace is all we have to compute. The computation of the same trace but in the HF necessitates computing all the helicity projections of the right hand side of(16). Acknowledge that this is a long computation. So by using the SF, that is the second equation in (14) the computation is straightforward

$$2iTr(\rho_{-s,s}\gamma_\mu) = -\frac{2s}{m}\left|\vec{k}\right|\eta_{1\mu} - 2ig_{\mu 0} \qquad (18)$$

But preferring the HF instead, one has to compute four helicity components using the helicity projectors,(13) and then (16) before recovering the result. The four helicity components are

$$\begin{array}{l} Tr(\rho_{\lambda,\lambda}\gamma_\mu) = -\frac{i\lambda}{m}\left|\vec{k}\right|\eta^\lambda_\mu \\ Tr(\rho_{-\lambda,\lambda}\gamma_\mu) = -ig_{\mu 0} \end{array} \qquad (19)$$

Note that the vertex amplitude(18) involves the transverse vector $\eta_{1\mu}$ only, while the helicity components(19) involve in addition $\vec{\eta}_2$ and $\vec{s}_L$ ($\vec{k}\times\vec{s}_L = 0$) but as unessential intermediary steps in the computation as they do not appear in the final result. This clearly shows that the HF is not appropriate in the presence of transverse spin.
Let us compute a more involved process $e^+e^- \to \tilde{e}^+_R\tilde{e}^-_R$ of the creation of right-handed selectrons by electrons via exchange of photinos in the chiral case (*photons*,$Z-bosons$ and *zino* exchanges are not considered as this computation is just an illustration).The positron has momentum k along the z axis in the e^+e^-centre of mass system and spin projection $-s$ along the transverse direction η_1 (here the x-axis), while the electron has momentum k' and spin $-s$ along η'_1 opposite to η_1(this is the natural polarization of the e^+e^-system in the storage ring). The momentum of the selectron is $p = (p^0, \vec{p})$ with $\vec{p} = |\vec{p}|\,(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. We may replace the electron-positron system by the quark-antiquark system with transverse polarization inherited from the parent proton-antiproton system transversely polarized. For the electron-photino vertex, we use the vertex fixed by supersymmetry in the chiral case $\sqrt{2}\,e\bar{\tilde{\gamma}}\frac{(1+\gamma_5)}{2}e\tilde{e}*_R$.Now to compute the amplitude we have first to include the anti- particle into the formalism since so far we have considered only the projector involving particles. To do so we adopt the convention that the anti-particle spinor is related to the particle spinor by the relation[4]

$$v(k,\lambda) = -\lambda\gamma_5 u(k,-\lambda) \qquad (20)$$

Applying the relation(20) to the spinor $u(k,s)|_{\vec{\zeta}=\vec{\eta}_1}$ we get

$$\begin{array}{rcl} u(k,s)|_{\vec{\zeta}=\vec{\eta}_1} & = & \frac{u(k,1)+su(k,-1)}{\sqrt{2}} \\ & = & s\gamma_5\frac{v(k,1)-sv(k,-1)}{\sqrt{2}} \\ & = & s\gamma_5 v(k,-s)|_{\vec{\zeta}=\vec{\eta}_1} \end{array} \qquad (21)$$

In what follows all quantities will be computed using the energy projectors $\not{k} + m$ (suitable for the massless limit or high energy which we will adopt here).
The amplitude computed at vanishing azimuthal angle is of the form

$$\begin{array}{l} 2e^2\bar{v}(k,-s)\frac{(1-\gamma_5)}{2}(\frac{\not{k}'-\not{p}+m_{\tilde{\gamma}}}{t-m_{\tilde{\gamma}}^2})\frac{(1+\gamma_5)}{2}u(k',-s) \\ = \frac{se^2}{2(t-m_{\tilde{\gamma}}^2)}Tr(1-\gamma_5)(\not{k}'-\not{p})\gamma_0\not{k}(1+s\gamma^5\not{\eta}_1\,) \end{array} \tag{22}$$

with $t = (k'-p)^2$. The $\gamma_5$term is vanishing as it is proportional to $\vec{p}.(\vec{k}' \times \vec{\eta}_1) = \vec{p}.\vec{\eta}_2 = 0$while the remaining term is

$$\begin{array}{l} \frac{e^2}{2(t-m_{\tilde{\gamma}}^2)}Tr(\not{k}'-\not{p})\gamma_0\not{k}\not{\eta}_1 \\ = \frac{2e^2k_0^2\beta_R\sin\theta}{(t-m_{\tilde{\gamma}}^2)} \end{array} \tag{23}$$

with $\beta_R = \frac{|\vec{p}|}{p^0}$ the velocity of the selectron. This is a one-step computation in the SF. To compute the same amplitude but in the HF as is commonly practiced so far, we have to compute four helicity amplitudes separately. The amplitude of the above process computed in the HF involves the quantities

$$\begin{array}{l} A_\mu = \bar{u}(k',\lambda')\gamma_\mu v(k,\lambda) \\ B_\mu = \bar{u}(k',\lambda')\gamma_\mu\gamma_5 v(k,\lambda) \end{array} \tag{24}$$

Computed in our convention, they lead to the expressions

$$\begin{array}{l} A_{\mu,\lambda,\lambda'} = \begin{cases} -i2mg_{\mu i}\hat{k}'^i & \lambda = \lambda' \\ -i2k_0'\lambda'\eta_\mu^\lambda & \lambda = -\lambda' \end{cases} \\ B_{\mu,\lambda,\lambda'} = \begin{cases} -i2\left|\vec{k}'\right|\eta_\mu^\lambda & \lambda = -\lambda' \\ -i2m\lambda' g_{\mu 0} & \lambda = \lambda' \end{cases} \end{array} \tag{25}$$

The helicity amplitudes turn out to all vanish in the limit of vanishing electron mass, except the one associated to the right-handed electron which we write explicitly

$$\begin{array}{l} 2e^2\bar{v}(k,-1)\frac{(1-\gamma_5)}{2}(\frac{\not{k}'-\not{p}+m_{\tilde{\gamma}}}{t-m_{\tilde{\gamma}}^2})\frac{(1+\gamma_5)}{2}u(k',1) \\ = \frac{e^2(k'-p)^\mu}{t-m_{\tilde{\gamma}}^2}(A^*_{\mu,1,-1} + B^*_{\mu,1,-1}) \\ = -ie^2\frac{4k_0p.n_\mu^\lambda}{t-m_{\tilde{\gamma}}^2} \\ = -ie^2\beta_R\frac{4k_0^2\sin\theta}{t-m_{\tilde{\gamma}}^2} \end{array} \tag{26}$$

Now using the decomposition of the spin projector $\tilde{\rho}_{s,s'} = u(k,s)\bar{v}(k',s')$ along the helicity projectors analogous to(16)

$$\tilde{\rho}_{-s,-s} = -\frac{i}{2}\left[s(\tilde{\rho}_{1,1} - \tilde{\rho}_{-1,-1}) + \tilde{\rho}_{-1,1} - \tilde{\rho}_{1,-1})\right] \tag{27}$$

We recover the transversely polarized amplitude which is according to(27)

$$\begin{aligned} \tfrac{i}{2}Tr(\tilde{\rho}_{1,-1}\cdots) &= \tfrac{i}{2}(-ie^2\beta_R\tfrac{4k_0^2\sin\theta}{t-m_{\tilde{\gamma}}^2}) \\ &= \tfrac{2e^2k_0^2\beta_R\sin\theta}{(t-m_{\tilde{\gamma}}^2)} \end{aligned} \tag{28}$$

The result is recovered but at the price of working out four helicity amplitudes instead of a unique amplitude in the case of the SF. The number of helicity amplitudes even increases by increasing the number of spinning particles in the process, such as for instance $e^+e^- \to \gamma\gamma$ which involves sixteen helicity amplitudes $T^{\lambda\lambda'}_{h\,h'}$.

4.2. The quark dipole magnetic moment

By using the Gordon decomposition we divide the dipole magnetic moment expression into two terms: one is the convection current part and the other is the spin part. As an application of our formalism we compute the convection part as it is the only part which involves the generalized spin density. Let us first show that the spin current part of the dipole magnetic moment does not involve the generalized spin density. The spin current part is proportional to

$$\int \left[\vec{\nabla}_q \times (Tr(q_\nu \vec{\sigma}^\nu \psi(k)\bar{\psi}(k^{'}))\right]\Big|_{\vec{q}=0} \frac{d^3k}{(2\pi)^3} \tag{29}$$

There are two terms resulting from the differentiation with respect to $\vec{q}$. The first one leads to the usual projector $\psi(k,s)\bar{\psi}(k,s)$ after setting $\vec{q}=0$, while the second one is proportional to $\vec{q}$ hence vanishing at $\vec{q}=0$. It then follows that the generalized projector ρ is effectively absent in the expression(29). The convection current part has the form

$$-i\int \left[Tr\vec{\nabla}_k\rho\right]_{\substack{\vec{k}'=\vec{k}\\ \vec{\zeta}'=\vec{\zeta}}} \times \vec{k}\frac{d^3k}{(2\pi)^3} \tag{30}$$

In computing the above expression we use the identities $\vec{\nabla}_k k_0 = \frac{\vec{k}}{k_0}$ and $\vec{\nabla}_k|\vec{k}| = \frac{\vec{k}}{|\vec{k}|}$ to eliminate all differentiations leading to terms proportional to $\vec{k}$ as this leads to $\vec{k}\times\vec{k}=0$ in (30). In particular any differentiation of the spin variable is eliminated as $\not{s}(\not{s}')$ is function of the boost parameter

$\omega=-\tanh^{-1}(\frac{|\vec{k}|}{k_0})(\omega')$ and hence necessarily leads to $\vec{k}$.With these remarks only the variables $\not k$and$\not k'$ are differentiated and we get

$$\begin{aligned} & \tfrac{i}{2(k_0+m)}\int(\vec{k}\times Tr\left[(\tfrac{\not k+m}{2m})(\tfrac{1+\gamma_5\not s}{2})(\gamma_0\vec{\gamma})\right])\tfrac{d^3k}{(2\pi)^3} \\ & =\tfrac{-i}{8m(k_0+m)}\int(\vec{k}\times Tr\gamma_5\not k\not s\gamma_0\vec{\gamma})\tfrac{d^3k}{(2\pi)^3} \\ & =-\tfrac{1}{2m(k_0+m)}\int|\vec{k}|^2\vec{s}_\perp(k)\tfrac{d^3k}{(2\pi)^3} \end{aligned} \tag{31}$$

Note the natural occurrence of the transverse spin $\vec{s}_\perp$ in the calculation of the convection current and hence there is room for the HF. Only the few steps in(31) are needed in the computation of the dipole moment and this has to be compared with the lengthy computation of the same observable using the Dirac spinors[5]

5. Conclusion

We constructed the SF in its version where the amplitude is expressed as a trace over Dirac indices. Such formalism is appropriate to processes involving transversity knowing that such processes are now of great popularity. A similar formalism but where the helicity is the principal entity already exits but it is more appropriate to processes involving states of definite helicity. Although the HF may also deal with processes with transversity too, it does so but only at the price of computing several helicity amplitudes.We also have used the SF to retrieve all formulas of the HF in an intuitive and clear way. On the other hand we have computed some processes with transverse polarizations, in both formalisms to illustrate the performance of the spin over the helicity in such processes. Finally the dipole magnetic moment as an example is shown to be exclusively expressed in term of spin and not of helicity.

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